Daniel Peralta-Salas
Instituto de Ciencias Matemáticas, Madrid

A problem of Berry and knotted zeros in the eigenfunctions of the harmonic oscillator

In 2001, motivated by finding eigenfunctions of the hydrogen atom whose nodal sets form torus knots, the theoretical physicist Michael Berry conjectured the same was true for the harmonic oscillator and asked whether any finite link could be realized as the nodal set of an eigenfunction to some quantum system. In this talk we give a proof that answers both these questions in the affirmative. Specifically, we show that any finite link in Euclidean space can be realized (up to a global diffeomorphism) as the union of connected components of the nodal set of a harmonic oscillator eigenfunction. We will see that the high energy asymptotics of the harmonic oscillator's eigenfunctions and the wealth of different solutions to the Helmholtz equation play key roles in this proof. Time permitting, I will also show that an analogous result holds for eigenfunctions of the hydrogen atom, which matches Berry's original setting. This is joint work with Alberto Enciso and David Hartley.